Telecom Paris ACCQ204, Coding Theory

# ASSIGNMENT 1

**Exercise 1** (RAID, distributed storage)**.** Redundant Arrays of Independent Disks consist of a set of disks such that any subset of $s$ disks can be disabled and the others are still able to reconstruct any requested file (the system can tell which disks are disabled).The rate of a RAID system corresponds to the rate at which data is stored.

1. Design a RAID system for 7 disks and $s = 2$. To do this you may want to consider the $(7, 4)$ Hamming code.

2. What happens if we use this code and try correct 3 erasures?

**Exercise 2.** Let $C$ be a code with minimum distance $d$. Prove that $C$ can correct any pattern of $e_1$ errors and $e_2$ erasures provided that $2e_1 + e_2 + 1 \leq d$. (Hint: given an erasure pattern, consider the code obtained by the deleting the erasure positions.)

**Exercise 3** (Best decoder)**.** Consider a set of $\mathcal{M}$ messages. A random message $M$ is chosen with probability $P(M = m) = p_m$ (hence $\sum_m p_m = 1$), encoded, and sent across a channel. Upon observing the channel output $y$, the receiver declares one of the messages by means of a decoder which maps each channel output to one of the messages. Let $D^*$ be the Maximum A Posteriori (MAP) decoding rule, i.e.

$$D^*(y) = \arg\max_m P(m|y).$$

1. Show that among all decoding functions, $D^*$ minimizes the error probability given any channel output.

2. Deduce that $D^*$ minimizes the average error probability among all decoding function.

**Exercise 4** (MAP decoder)**.** Consider communication over a binary symmetric channel with crossover probability $p$. There are two possible equally likely messages that are encoded over three bits: $000$ and $111$. What is the error probability of the MAP decoder?

**Exercise 5** ($A(n, d, w), A(n, d)$)**.** For any integers $n, d, w$ let $A(n, d, w)$ be the largest possible size of a set of binary vectors of length $n$ and weight $w$ whose minimum distance is at least $d$, and let $A(n, d)$ be the largest possible size of a set of length $n$ binary vectors whose minimum distance is at least $d$. Prove that

$$A(n, d) \leq \sum_{w=0}^{n} A(n, d, w)$$